Pymble Ladies' College

Mathematics Advanced
HSC Trial Examination
Term 3 2022

General Instructions	• Reading time – 10 minutes • Working time – 3 hours • Write using non-erasable black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • For questions in Section II, show relevant mathematical reasoning or calculations
Total marks 100	**Section 1 – 10 marks** (pages 1-4) • Attempt Questions 1-10 • Allow about 15 minutes for this section **Section II – 90 marks** (pages 6-27) • Attempt Questions 11-33 • Allow about 2 hours and 45 minutes for this section

BLANK PAGE

SECTION I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer sheet for Questions 1-10

---

1 The function $f(x) = \frac{1}{x}$ is translated 3 units up and 2 units right to produce $y = g(x)$.

Which of the following is the equation of the translated function $g(x)$?

(A) $g(x) = \frac{1}{x+2} - 3$

(B) $g(x) = \frac{1}{x+2} + 3$

(C) $g(x) = \frac{1}{x-2} + 3$

(D) $g(x) = \frac{1}{x-2} - 3$

2 A function is given by $f(x) = \begin{cases} \frac{3x^2}{125} & 0 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$

If this function is a continuous probability distribution, what is the area under the curve?

(A) -1

(B) 0.5

(C) 1

(D) 2

3 What is the period of the function $f(x) = \sin\left(3x - \frac{\pi}{3}\right)$, where x is real?

(A) $\frac{\pi}{9}$

(B) $\frac{2\pi}{3}$

(C) 2π

(D) $\frac{\pi}{3}$

4 Which of the following is the gradient of the normal to $y = \log_2 x$ at the point $(8,3)$?

(A) $-\frac{1}{8\ln 2}$

(B) $-8\ln 2$

(C) $\frac{1}{8\ln 2}$

(D) $8\ln 2$

5 A function is defined by the rule

$$f(x) = \begin{cases} 1 & \text{for } x < 1 \\ x+2 & \text{for } x \geq 1 \end{cases}$$

Which statement is **incorrect**?

(A) The value of $f(-2)$ is 1.

(B) The graph is not continuous at $x = 1$.

(C) The domain is all real values of x.

(D) The range is $f(x) \geq 1$.

6 The amount of water that Eleanor uses to wash her car is normally distributed with a mean of 50 litres and a standard deviation of 4 litres.
On what percentage of occasions would Eleanor expect to use between 42 litres and 46 litres of water to wash her car?

(A) 13.5%

(B) 27%

(C) 34%

(D) 68%

7 What is the exact value of $\int_0^4 \sqrt{16-x^2}\ dx$?

(A) 4

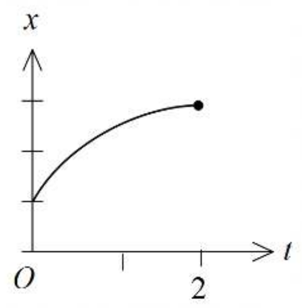
(B) 16π

(C) 2

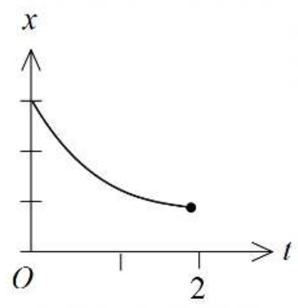
(D) 4π

8 A particle is moving in a straight line. For $0 \le t \le 2$, its velocity is positive and its acceleration is negative.
Which graph best represents the displacement function $x(t)$ of this particle?

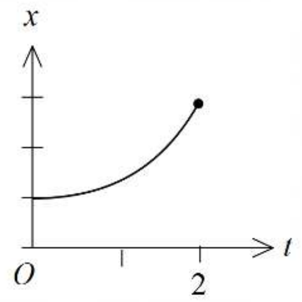
(A)



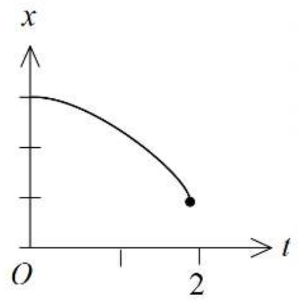
(B)



(C)



(D)



9 For events A and B over a sample space, $P(A \cap B) = 0.2$ and $P(A \mid B) = 0.25$.
What is $P(B)$?

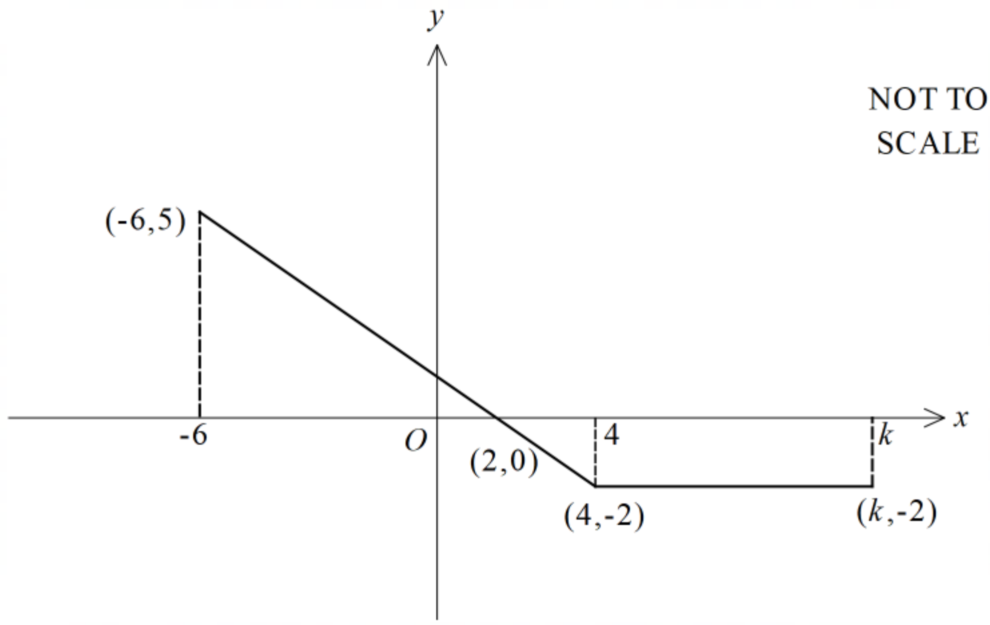
(A) 0.45

(B) 0.05

(C) 0.75

(D) 0.8

10



By using the graph above, which value of *k* satisfies

$$\int_{-6}^{k} f(x)\,dx = 0 \,?$$

(A) 6

(B) 13

(C) 11

(D) 12

Name………………………………..

Teacher's Name……………………..

Mathematics Advanced
Section II

Answer Booklet

90 marks
Attempt Questions 11-33
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer in the spaces provided. These spaces provide guidance for the expected length of the answer.
  Extra writing space is provided at the end of the paper.
- Your responses should include relevant mathematical reasoning and/or calculations

Question 11 (2 marks)

What is the domain of the function $f\left(x\right)=\dfrac{1}{\sqrt{x^2-9}}$? **2**

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

Question 12 (3 marks)

A curve has the equation $y=\left(2x+3\right)e^{x^2}$. **3**

Find the x-coordinate(s) of any stationary point(s) on the curve.

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

Question 13 (2 marks)

If $\int_1^5 h\left(x\right)dx=6$, what is the value of $\int_1^5 \left(h\left(x\right)+2\right)dx$? **2**

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

Question 14 (6 marks)

Max did a survey of a group of people he knew about their age and how much they earn each week. The results are shown in the table below.

Age (years) (x)	18	45	28	15	32	68
Wage ($/week) ($W$)	715	2350	1530	438	1690	1320

(a) Using your calculator, find the correlation coefficient (r) and explain what type and strength of correlation this data gives. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) Using your calculator write down the equation of the least-squares regression line in the form $W = Bx + A$ where A and B are integers. **1**

……………………………………………………………………………………………

(c) Use your equation to estimate the earnings of a 50 year-old worker. **1**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(d) Could your equation from part (b) be used to make valid estimates for ages greater than 68 and less than 15 years? **2**
Validate your response with calculations and or reasons.

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 15 (2 marks)

Find $\int \left(e^{4x} + 4\sqrt{x}\right) dx.$ **2**

……………………………………………………………………………………………

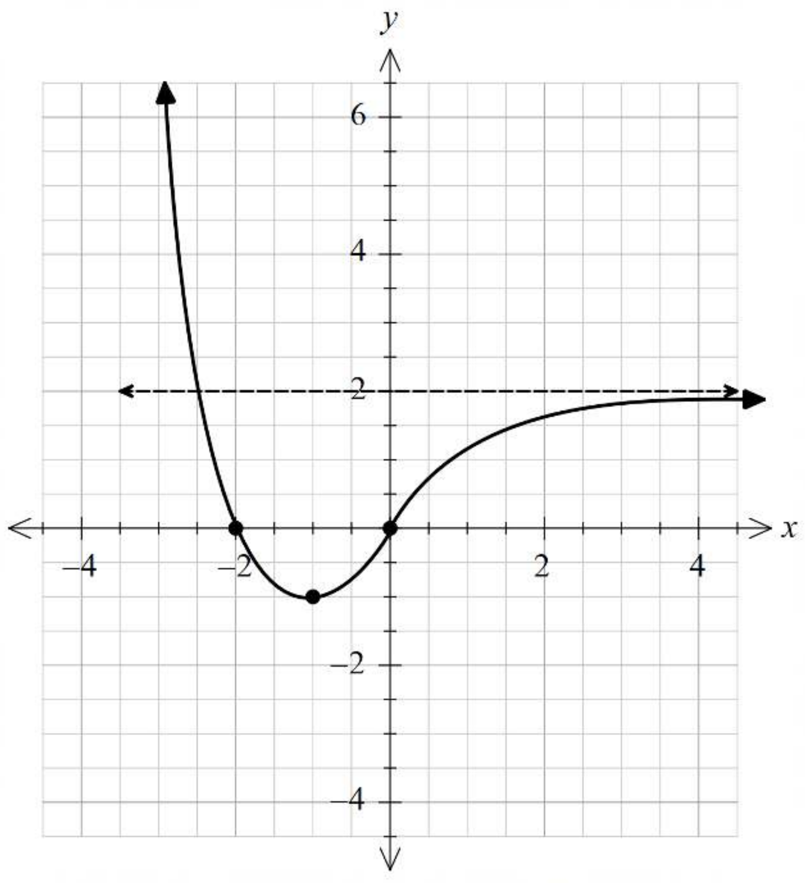
……………………………………………………………………………………………

……………………………………………………………………………………………

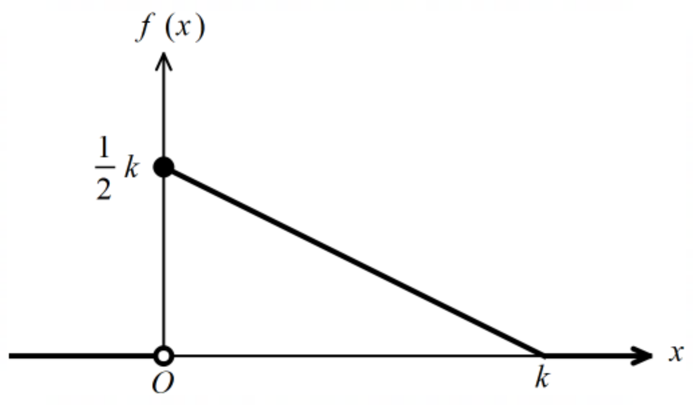
……………………………………………………………………………………………

Question 16 (3 marks)

The curve of $y = f(x)$ is given below. Sketch $y = -2f(-2x)$, on the same set of axes, indicating all key features of your sketch including intercepts, turning points and any asymptotes. The dashed line represents an asymptote and there is a turning point at $(-1, -1)$. **3**



Question 17 (6 marks)



The diagram shows the graph of the probability density function, *f,* of a random variable *X*.

(a) Find the value of the constant *k*. **2**

....

....

....

....

....

(b) Using this value of *k*, find the equation of $f(x)$ for $0 \le x \le k$. **1**

....

....

....

....

....

Question 17 continues on next page

Question 17 continued.

(c) Find the value of p such that $\mathrm{P}\left(p < X < 1\right) = 0.25$. **3**

....

....

....

....

....

....

....

....

....

....

....

....

....

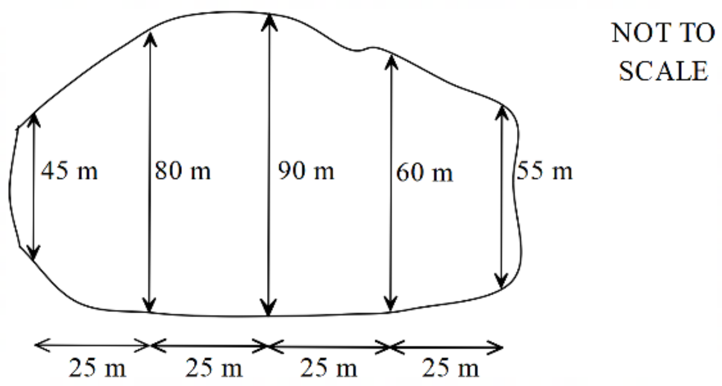
....

....

....

Question 18 (7 marks)

A factory located next to a lake has recently been shut down due to concerns about chemical run off. A bird's eye view of the lake is given in the diagram below.



Local volunteers have begun a clean-up effort to remove these harmful chemicals from the lake. The concentration of chemicals is given by the equation:

$$C = 1.5 - 0.4e^{kt}$$

Let C be the concentration of chemicals in the lake in kg/m^3, where k is a constant and t is the number of years after the clean-up effort has started.

(a) The initial weight of chemical run off in the lake is 38500 kg, and the average depth of the lake is 5 m. Use the trapezoidal rule, with five function values, to show that the initial concentration of chemical run off can be estimated as 1.1 kg/m^3. **3**

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

Question 18 continues on next page

Question 18 continued.

(b) Find the value of k to three significant figures if it takes 2.45 years to remove all chemical run off from the lake. **2**

....

....

....

....

....

....

....

....

(c) Find the rate of change in the concentration of chemical run off in the lake at $t = 1.8$ years. Round your answer to two decimal places. **2**

....

....

....

....

....

....

....

....

....

....

Question 19 (2 marks)

Find the possible values of the constant c for which the line $y = c$ is a tangent to the curve $y = 5\sin\frac{x}{3} + 4$. **2**

....

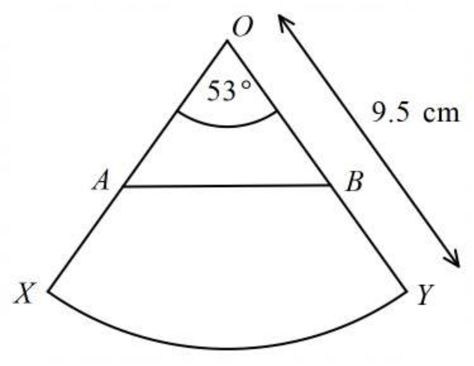
....

....

....

....

Question 20 (4 marks)



The diagram shows a sector *OXY* of a circle with centre *O* and radius 9.5 cm.
The sector angle is $53°$. *A* lies on *OX*, *B* lies on *OY* and $OA = OB$.

(a) Show that the area of the sector is 41.7 cm^2, correct to 1 decimal place. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) The area of triangle *OAB* is $\frac{1}{3}$ of the area of sector *OXY*. **2**

Calculate *OA*. correct to 1 decimal place.

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 21 (3 marks)

Solve the equation $\ln\left(x^3 - 3\right) = 3\ln x - \ln 3$. **3**

Give your answer correct to 3 significant figures.

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

Question 22 (3 marks)

There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random.

(a) Find the probability that the student prefers swimming. **1**

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

(b) Determine whether the events "the student is male" and "the student prefers swimming" are independent, justifying your answer with mathematical reasoning. **2**

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

Question 23 (5 marks)

A geometric series is such that the sum of the first 4 terms is 17 times the sum of the first 2 terms. It is given that the common ratio of this geometric series is positive and not equal to 1.

(a) Find the common ratio of this geometric series. **3**

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

(b) Given that the 6^{th} term of the geometric series is 64, find the first term. **1**

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

(c) Explain mathematically why this geometric series does not have a sum to infinity. **1**

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

Question 24 (2 marks)

Show that $\dfrac{2\cos\theta\sin^2\theta + 2\cos^3\theta}{4\sin\theta} = \dfrac{1}{2}\cot\theta.$ **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 25 (3 marks)

The temperature of a freshly served bowl of pho bo from Tan Viet Noodle House is given by the following equation:

$$T = 22 + 60e^{-0.1t}$$

where T is the temperature in degrees Celsius and t is the time in minutes.

(a) To the nearest degree, what is the temperature of the pho after 1 minute? **1**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) How long, to the nearest minute, will it take for the temperature of the pho to drop to 62°? **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 26 (5 marks) **5**

Kate and David are buying a house for \$1 700 000. They have a \$200 000 deposit and will need to borrow the remaining balance.

An interest rate of 3.6% p.a. compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly payments (M) over a 30 year period.

How much should Kate and David be paying each month to fully pay off the house in the 30 year period and how much interest do they pay over the life of the loan?

Question 27 (5 marks)

Use the following standard z-table for this question.

z	First decimal place .0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

(a) What is $P(z \geq 1.8)$? **1**

....

....

....

....

(b) What is $P(-1.2 \leq z \leq 0.3)$? **2**

....

....

....

....

....

(c) The birth weight of babies is known to be normally distributed. According to data which covers Australian pregnancies between 1998 and 2007, the mean μ birthweight for boys was 3632 grams with a standard deviation σ of 430 grams. Using the table above, what is the approximate probability that a randomly selected newborn boy will weigh less than 3890 grams? **2**

....

....

....

....

....

....

....

Question 28 (7 marks)

Consider the curve $y = 4x^2 - 2x^3$.

(a) Find the stationary points of the curve $y = 4x^2 - 2x^3$. Determine their nature. **3**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Question 28 continues on next page

Question 28 continued

(b) Show that there is an inflection point on the curve. **1**

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

(c) For what interval is the curve $y = 4x^2 - 2x^3$ increasing? **1**

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

Question 28 continues on next page

Question 28 continued.

(d) Hence, sketch the graph of the curve $y = 4x^2 - 2x^3$. Clearly label the stationary points, the point of inflection and any intercepts with the axes. **2**

Question 29 (3 marks)

(a) Differentiate $y = \frac{\ln x}{x}$. **1**

....

....

....

....

....

....

....

(b) Hence, evaluate $\int_e^{e^2} \frac{2 - \ln x^2}{x^2}\, dx$. **2**

....

....

....

....

....

....

....

....

....

....

....

Question 30 (4 marks)

Consider the probability density function f for a random variable X given by

$$f(x)=\begin{cases}\dfrac{a}{\sqrt{9+2x}} & 0\le x\le 8\\[2ex] 0 & \text{otherwise}\end{cases}$$

where a is a constant.

(a) Find the value of a. **2**

....

....

....

....

....

....

....

....

....

....

....

(b) Find the cumulative distribution function, $F(x)$. **2**

....

....

....

....

....

....

....

....

....

....

Question 31 (4 marks)

A particle moves along a straight line so that its displacement x metres to the right of a fixed point O is given by

$$x = 12\ln(t+2) - 2t + 5,$$

where the time t is measured in seconds.

(a) What is the initial position of the particle? Give your answer in exact form. **1**

....

....

(b) Find the expression for the velocity of the particle at time t. **1**

....

....

....

....

(c) Find the time when the particle is at rest. **1**

....

....

....

....

....

(d) What happens to the acceleration eventually? **1**

....

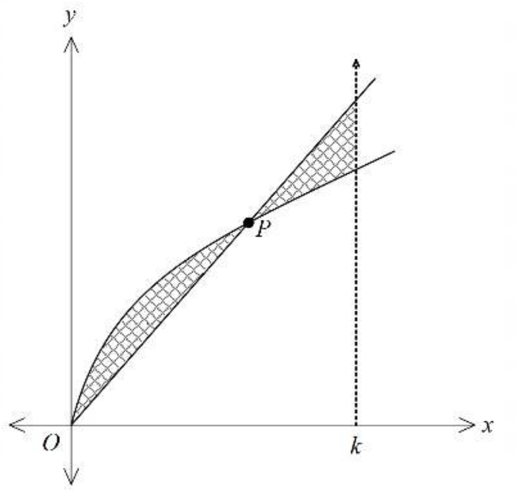
....

....

....

Question 32 (4 marks)

The diagram below shows the area bounded between two curves, $y = \sqrt{x}$ and $y = x$.



(a) P is the point of intersection of the curves. Write down the coordinates of P. **1**

……………………………………………………………………………………

……………………………………………………………………………………

Consider the region bounded by the curves, the line $x = 0$ and $x = k$, where $k > 1$.

(b) For what value of k will the shaded area to the left of point P be equal to the shaded area to the right of P? **3**

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

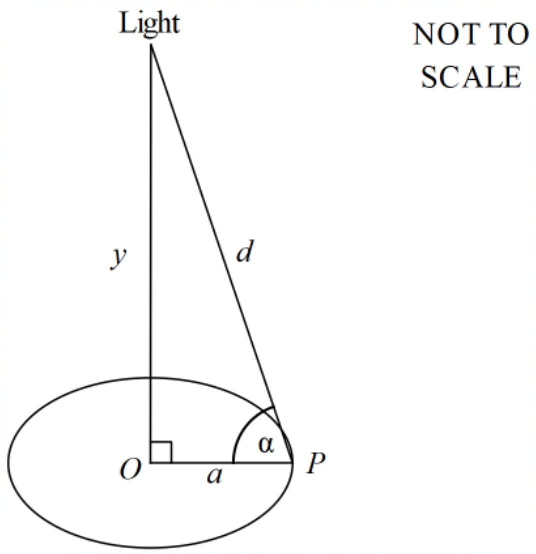
……………………………………………………………………………………

……………………………………………………………………………………

Question 33 (5 marks)

A light is to be placed over the centre of a circle, radius a units. The intensity I of the light is proportional to the sine of the angle α at which the rays strike the circumference of the circle, divided by the square of the distance d from the light to the circumference of the circle,

i.e. $I = \dfrac{k \sin \alpha}{d^2}$, where k is a positive constant.



(a) Show that $I = \dfrac{ky}{\left(y^2 + a^2\right)^{\frac{3}{2}}}$. **2**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Question 33 continues on next page

Question 33 continued

(b) Find the best height for the light to be placed over the centre of the circle in order to provide maximum illumination to the circumference. **3**

END OF PAPER

If you use this space, clearly indicate which question you are answering.

If you use this space, clearly indicate which question you are answering.

……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..

BLANK PAGE

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

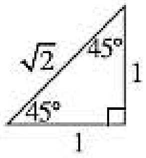
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

$A = \frac{1}{2}ab\sin C$



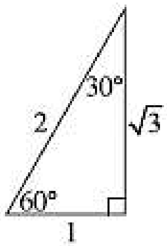
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \frac{1}{2}r^2\theta$



Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$\cos A = \frac{1-t^2}{1+t^2}$

$\tan A = \frac{2t}{1-t^2}$

$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

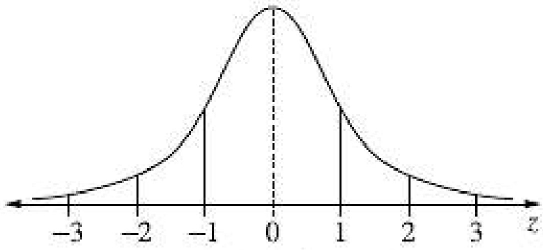
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \le r) = \int_a^r f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$= \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta) = r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^{2}(x - c)$$

Pymble Ladies' College

Mathematics Advanced
HSC Trial Examination
Term 3 2022

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning or calculations

Total marks 100

Section 1 – 10 marks (pages 1-4)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6-27)

- Attempt Questions 11-33
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10

---

1 The function $f(x)=\frac{1}{x}$ is translated 3 units up and 2 units right to produce $y=g(x)$.

Which of the following is the equation of the translated function $g(x)$?

(A) $g(x)=\frac{1}{x+2}-3$

(B) $g(x)=\frac{1}{x+2}+3$

(C) $g(x)=\frac{1}{x-2}+3$

(D) $g(x)=\frac{1}{x-2}-3$

2 A function is given by $f(x)=\begin{cases}\frac{3x^2}{125} & 0\le x\le 5\\ 0 & \text{elsewhere}\end{cases}$

If this function is a continuous probability distribution, what is the area under the curve?

(A) -1

(B) 0.5

(C) 1

(D) 2

3 What is the period of the function $f(x)=\sin\left(3x-\frac{\pi}{3}\right)$, where x is real?

(A) $\frac{\pi}{9}$

(B) $\frac{2\pi}{3}$

(C) 2π

(D) $\frac{\pi}{3}$

4 Which of the following is the gradient of the normal to $y = \log_2 x$ at the point $(8,3)$?

(A) $-\dfrac{1}{8\ln 2}$

(B) $-8\ln 2$

(C) $\dfrac{1}{8\ln 2}$

(D) $8\ln 2$

5 A function is defined by the rule

$$f(x) = \begin{cases} 1 & \text{for } x < 1 \\ x+2 & \text{for } x \geq 1 \end{cases}$$

Which statement is **incorrect**?

(A) The value of $f(-2)$ is 1.

(B) The graph is not continuous at $x = 1$.

(C) The domain is all real values of x.

(D) The range is $f(x) \geq 1$.

6 The amount of water that Eleanor uses to wash her car is normally distributed with a mean of 50 litres and a standard deviation of 4 litres.
On what percentage of occasions would Eleanor expect to use between 42 litres and 46 litres of water to wash her car?

(A) 13.5%

(B) 27%

(C) 34%

(D) 68%

7 What is the exact value of $\int_0^4 \sqrt{16-x^2}\, dx$?

(A) 4

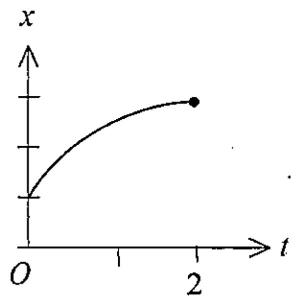
(B) 16π

(C) 2

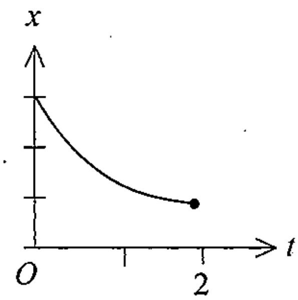
(D) 4π

8 A particle is moving in a straight line. For $0 \le t \le 2$, its velocity is positive and its acceleration is negative.
Which graph best represents the displacement function $x(t)$ of this particle?

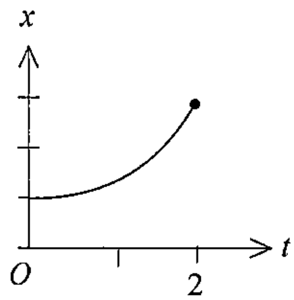
(A)



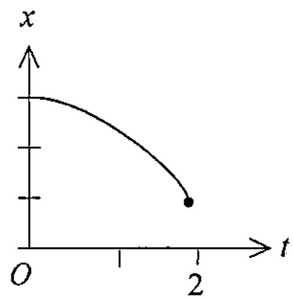
(B)



(C)



(D)



9 For events A and B over a sample space, $P(A \cap B) = 0.2$ and $P(A \mid B) = 0.25$.
What is $P(B)$?

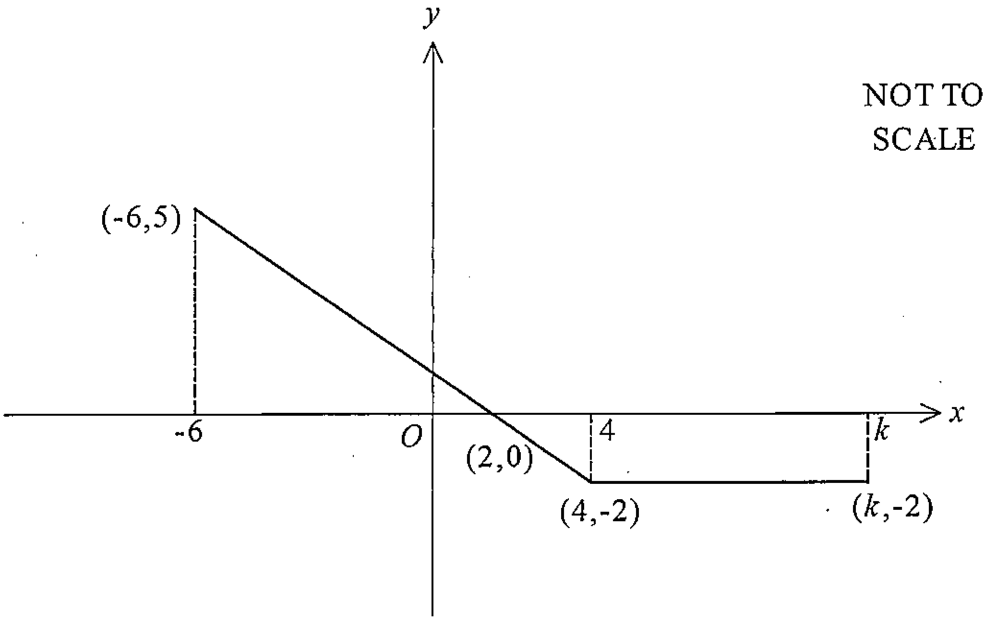
(A) 0.45

(B) 0.05

(C) 0.75

(D) 0.8

10



By using the graph above, which value of k satisfies

$$\int_{-6}^{k} f(x)\,dx = 0\,?$$

(A) 6

(B) 13

(C) 11

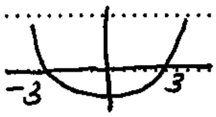
(D) 12

Question 11 (2 marks)

What is the domain of the function $f(x)=\frac{1}{\sqrt{x^2-9}}$? **2**

$x^2-9>0$

Domain: $x \in (-\infty,-3) \cup (3,\infty)$



Bald answer → 0

Correct answer incorrect working → 1

2	Correct
1	Correct inequality and work towards a sol^n

Question 12 (3 marks)

A curve has the equation $y=(2x+3)e^{x^2}$. **3**

Find the x-coordinate(s) of any stationary point(s) on the curve.

$dy/dx = (2x+3) \times 2xe^{x^2} + e^{x^2} \times 2$

$= 2xe^{x^2}(2x+3) + 2e^{x^2}$

When $dy/dx = 0$ $\quad 2e^{x^2}(x(2x+3)+1)=0$

$2e^{x^2}=0$ or $2x^2+3x+1=0$

No sol^n or $(2x+1)(x+1)=0$

$\therefore x=-1/2$ or $x=-1$

3	Correct
2	Correct derivative and factorisation with work toward sol^n
1	Correct derivative

Question 13 (2 marks)

If $\int_1^5 h(x)\,dx = 6$, what is the value of $\int_1^5 (h(x)+2)\,dx$? **2**

$\int_1^5 h(x)\,dx + \int_1^5 2\,dx$

$= 6 + 2[x]_1^5$

$= 6 + 2(5-1)$

$= 14$

2	Correct
1	Breaking integral apart and progress to sol^n

Question 14 (6 marks)

Max did a survey of a group of people he knew about their age and how much they earn each week. The results are shown in the table below.

Age (years) (x)	18	45	28	15	32	68
Wage ($/week) ($W$)	715	2350	1530	438	1690	1320

(a) Using your calculator, find the correlation coefficient (r) and explain what type and strength of correlation this data gives. 2

$r = 0.526321713$

positive, moderate strength correlation

2	Both correct
1	Correct part

(b) Using your calculator write down the equation of the least-squares regression line in the form $W = Bx + A$ where A and B are integers. 1

$W = 18.47948276x + 706.0377586$

1 r/w

(c) Use your equation to estimate the earnings of a 50 year-old worker. 1

$W = 18.47948276 \times 50 + 706.0377586$

$= \$1630.01$

1 r/w

(d) Could your equation from part (b) be used to make valid estimates for ages greater than 68 and less than 15 years?
Validate your response with calculations and or reasons. 2

The correlation is only moderately positive ∴ extrapolation would not result in an accurate estimate

People below 15 and over 68 would not be working, too young, retired. This equation does not account for this.

2	Correct two reasons
1	Reference to Correlation Coefficient or reference to < 15 > 68 as not working or retired

Question 15 (2 marks)

Find $\int \left(e^{4x} + 4\sqrt{x}\right) dx$. **2**

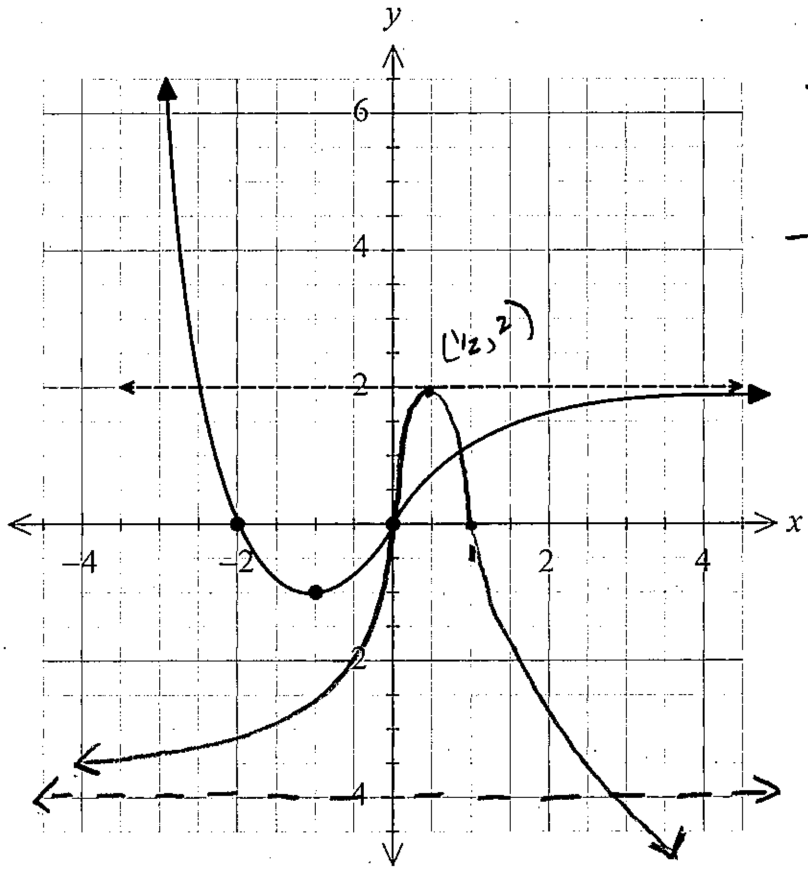
$= \frac{e^{4x}}{4} + \frac{8x^{3/2}}{3} + C$

$= \frac{1}{4}e^{4x} + \frac{8}{3}\sqrt{x^3} + C$

2	correct
1	one part of integral correct.

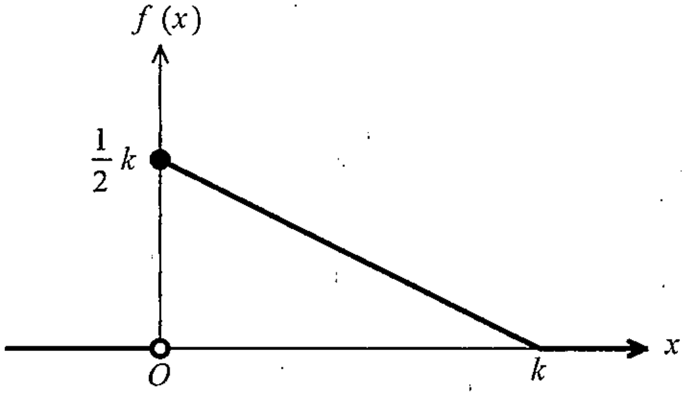
Question 16 (3 marks)

The curve of $y = f(x)$ is given below. Sketch $y = -2f(-2x)$, on the same set of axes, indicating all key features of your sketch including intercepts, turning points and any asymptotes. The dashed line represents an asymptote and there is a turning point at $(-1,-1)$. **3**



3	correct
2	Assymptote turning point correct, one error
1	Turning point or asymptote correct.

Question 17 (6 marks)



The diagram shows the graph of the probability density function, f, of a random variable X.

(a) Find the value of the constant k. **2**

Area under the curve = 1

$\frac{1}{2} \times \frac{1}{2} k \times k = 1$

$k^2 = 4$

$k = 2$; $k > 0$

2 | Correct
1 | Correct eq^n

(b) Using this value of k, find the equation of $f(x)$ for $0 \le x \le k$. **1**

y-intercept $(0, 1)$

x-intercept $(2, 0)$

$\therefore m = -\frac{1}{2}$

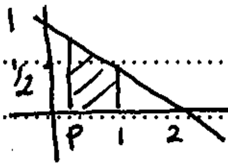
$\therefore f(x) = \begin{cases} -\frac{1}{2}x + 1 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

1 | Must have domain

Question 17 continues on next page

Question 17 continued.

(c) Find the value of p such that $\mathrm{P}(p < X < 1) = 0.25$. **3**



$\frac{1}{4} = \frac{1}{2}(1-p)\left(\frac{1}{2} - \frac{p}{2} + 1\right)$ Area of trapezium

$1 = 2(1-p)\left(\frac{3}{2} - \frac{p}{2}\right)$

$(1-p)(3-p) = 1$

$p^2 - 4p + 2 = 0$

$p = \frac{4 \pm \sqrt{16-8}}{2}$

$= 2 \pm \sqrt{2}$ $\quad p < 1$

$\therefore p = 2 - \sqrt{2}$

3	Correct
2	Correct area of a trapezium and progress to solⁿ
1	Correct area of trapezium.

OR

$\int_p^1 \left(-\frac{1}{2}x + 1\right) dx = \frac{1}{4}$

$\left[-\frac{x^2}{4} + x\right]_p^1 = \frac{1}{4}$

$\frac{3}{4} - \left(\frac{-p^2}{4} + p\right) = \frac{1}{4}$

$p^2 - 4p + 2 = 0$

$p = \frac{4 \pm \sqrt{16-8}}{2}$

$p = 2 \pm \sqrt{2}$

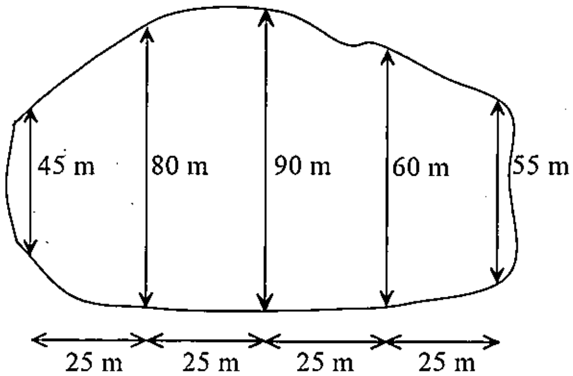
$0 < p < 1$

$\therefore p = 2 - \sqrt{2}$

3	Correct
2	1 error
1	Just integral equal to 0.25 with correct limits

Question 18 (7 marks)

A factory located next to a lake has recently been shut down due to concerns about chemical run off. A bird's eye view of the lake is given in the diagram below.



NOT TO SCALE

Local volunteers have begun a clean-up effort to remove these harmful chemicals from the lake. The concentration of chemicals is given by the equation:

$$C = 1.5 - 0.4e^{kt}$$

Let C be the concentration of chemicals in the lake in kg/m^3, where k is a constant and t is the number of years after the clean-up effort has started.

(a) The initial weight of chemical run off in the lake is 38500 kg, and the average depth of the lake is 5 m. Use the trapezoidal rule, with five function values, to show that the initial concentration of chemical run off can be estimated as 1.1 kg/m^3. **3**

0	25	50	75	100
45	80	90	60	55

$A \doteqdot \frac{1}{2} \times 25\left(45 + 55 + 2\left(80 + 90 + 60\right)\right)$ 1

$= 7000$

$V = 7000 \times 5$

$= 35000 \text{ m}^3$ 1

Run off $= \frac{38500}{35000}$

$= 1.1 \text{ kg/m}^3$ 1

3	Correct
2	Correct trapezoidal rule + Volume
1	Correct trapezoidal rule

Question 18 continues on next page

Question 18 continued.

(b) Find the value of k to three significant figures if it takes 2.45 years to remove all chemical run off from the lake. **2**

$1.5 - 0.4e^{2.45k} = 0$

$e^{2.45k} = \frac{1.5}{0.4}$

$2.45k = \ln\left(\frac{1.5}{0.4}\right)$

$k = \frac{1}{2.45}\ln\left(\frac{1.5}{0.4}\right)$

$= 0.539$

2	Correct
1	Correct eqn, 1 error

(c) Find the rate of change in the concentration of chemical run off in the lake at $t = 1.8$ years. Round your answer to two decimal places. **2**

$\frac{dC}{dt} = -0.4ke^{kt}$ 1

When $t = 1.8$

$\frac{dC}{dt} = -0.4 \times 0.539492179e^{0.539492179 \times 1.8}$ 1 [ISE]

$= -0.569878...$

$\doteqdot -0.57 \ kg/m^3$ (2dp)

2	correct
1	Calculation error

Question 19 (2 marks)

Find the possible values of the constant c for which the line $y = c$ is a tangent to the curve $y = 5\sin\frac{x}{3} + 4$. **2**

Amplitude is 5; Centre is 4

$\therefore c = 9$ or $c = -1$
1 1

2	Both correct
1	1 correct.

OR $y' = \frac{5}{3}\cos\frac{x}{3}$

$y' = 0$: $0 = \frac{5}{3}\cos\frac{x}{3}$ 1

$\frac{x}{3} = \frac{\pi}{2}, \frac{3\pi}{2}$

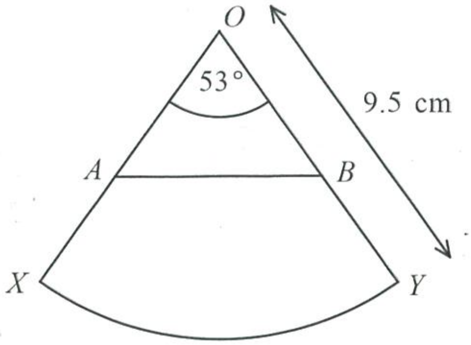
$x = \frac{3\pi}{2}, \frac{9\pi}{2}$

↓ ↓

$y = 9$ $y = -1$

Sol'n: $c = 9$ or $c = -1$ 1

Question 20 (4 marks)



The diagram shows a sector *OXY* of a circle with centre *O* and radius 9.5 cm. The sector angle is 53°. *A* lies on *OX*, *B* lies on *OY* and *OA* = *OB*.

(a) Show that the area of the sector is 41.7 cm^2, correct to 1 decimal place. **2**

$A = \frac{1}{2} \times 9.5^2 \times \frac{53\pi}{180}$ Radians | OR Degrees $A = \frac{53}{360} \times \pi \times 9.5^2$

$= 41.741173072$

$\doteqdot 41.7$ (1 dp) cm^2

2	Correct
1	1 error

(b) The area of triangle *OAB* is $\frac{1}{3}$ of the area of sector *OXY*. **2**

Calculate *OA*. correct to 1 decimal place.

Area $\triangle OAB = \frac{1}{3} \times \left(\frac{1}{2} \times 9.5^2 \times \frac{53\pi}{180}\right)$ (a) 1

(13.91339...)

$\frac{1}{6} \times 9.5^2 \times \frac{53\pi}{180} = \frac{1}{2} \times OA^2 \times \sin 53°$

$OA = 5.902898128$ 1

$\doteqdot 5.9$ cm (1 dp)

2	correct
1	Correct eq^n

Question 21 (3 marks)

Solve the equation $\ln\left(x^3 - 3\right) = 3\ln x - \ln 3$. **3**

Give your answer correct to 3 significant figures.

$\ln(x^3 - 3) = \ln \frac{x^3}{3}$ — ①

$\therefore x^3 - 3 = \frac{x^3}{3}$

$\frac{2x^3}{3} = 3$

$x = \sqrt[3]{9/2}$

$= 1.650963624$

$\doteq 1.65$ (3 sig. fig)

3	Correct
2	Contraction and removal of log 1 error
1	Correct line ①

Question 22 (3 marks)

There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random.

(a) Find the probability that the student prefers swimming. **1**

$P(\text{swimming}) = \frac{135}{400}$

$= \frac{27}{80}$

1 r/w

(b) Determine whether the events "the student is male" and "the student prefers swimming" are independent, justifying your answer with mathematical reasoning. **2**

$P(S) = \frac{27}{80}$

$P(M) = \frac{9}{20}$

$P(S \cap M) = \frac{31}{400}$ 1

$P(S) \times P(M) = \frac{27}{80} \times \frac{9}{20}$

1 $= \frac{243}{1600}$

$P(S \cap M) \neq P(S) \times P(M)$

$\therefore$ Events are not independent

2	Correct
1	Calculation of $P(S \cap M)$ and attempt using appropriate method

OR $P(M|S) = \frac{P(M \cap S)}{P(S)}$

$= \frac{31}{135}$

$P(M) = \frac{9}{20}$

$P(M|S) \neq P(M)$

Question 23 (5 marks)

A geometric series is such that the sum of the first 4 terms is 17 times the sum of the first 2 terms. It is given that the common ratio of this geometric series is positive and not equal to 1.

(a) Find the common ratio of this geometric series. **3**

$S_4 = 17 S_2$

$\frac{a(1-r^4)}{1-r} = \frac{17a(1-r^2)}{1-r}$

$a(1-r^4) = 17a - 17ar^2$

$1-r^4 = 17-17r^2$

$r^4 - 17r^2 + 16 = 0$ – ①

$(r^2-16)(r^2-1) = 0$

$\therefore r = \pm 4$ or $r = \pm 1$

But $r > 0$ and $r \neq 1$ $\therefore r = 4$

3	Correct incl. reasons for disreg.
2	Correct eqⁿ ① and progress to solve
1	$S_4 = \frac{a(1-r^4)}{1-r}$ OR $\frac{17a(1-r^2)}{1-r}$

(b) Given that the 6th term of the geometric series is 64, find the first term. **1**

$T_6 = ar^5$

$64 = 4^5 \times a$

$a = \frac{64}{4^5}$

$a = \frac{1}{16}$

1 r/w

(c) Explain mathematically why this geometric series does not have a sum to infinity. **1**

$r = 4$. For S_∞ to exist, $|r| < 1$

$\therefore S_\infty$ does not exist for this series

1 r/w

Question 24 (2 marks)

Show that $\dfrac{2\cos\theta\sin^2\theta + 2\cos^3\theta}{4\sin\theta} = \dfrac{1}{2}\cot\theta$. 2

LHS: $\dfrac{2\cos\theta(\sin^2\theta + \cos^2\theta)}{4\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$= \dfrac{\cos\theta}{2\sin\theta}$

$= \dfrac{1}{2}\cot\theta$; $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$=$ RHS

2 | Correct
1 | Factorisation and use of $\cos^2\theta + \sin^2\theta = 1$

Question 25 (3 marks)

The temperature of a freshly served bowl of pho bo from Tan Viet Noodle House is given by the following equation:

$$T = 22 + 60e^{-0.1t}$$

where T is the temperature in degrees Celsius and t is the time in minutes.

(a) To the nearest degree, what is the temperature of the pho after 1 minute? 1

$T = 22 + 60e^{-0.1}$

$= 76.29024508$

$\doteqdot 76^\circ C$

1/r/w

(b) How long, to the nearest minute, will it take for the temperature of the pho to drop to 62°? 2

$62 = 22 + 60e^{-0.1t}$

$e^{-0.1t} = 2/3$

$-0.1t = \ln(2/3)$

$t = \dfrac{-\ln(2/3)}{0.1}$

$= 4.05565\ldots$

$\doteqdot 4$ minutes

2 | correct at calc. display
1 | sub T=62 and attempts to solve

Question 26 (5 marks) 5

Kate and David are buying a house for \$1 700 000. They have a \$200 000 deposit and will need to borrow the remaining balance.

An interest rate of 3.6% p.a. compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly payments (M) over a 30 year period.

How much should Kate and David be paying each month to fully pay off the house in the 30 year period and how much interest do they pay over the life of the loan?

Amount borrowed = 1500000 $\frac{3.6}{12} = 0.3\%$ /month ①

30 years = 360 months Let M be the instalment

$A_1 = 1500000 \times 1.003 - M$

$A_2 = A_1 \times 1.003 - M$
$= (1500000 \times 1.003 - M) \times 1.003 - M$
$= 1500000 \times 1.003^2 - M(1.003 + 1)$ ①

⋮

$A_{360} = 0$

$\therefore 1500000 \times 1.003^{60} - M(1.003^{359} + \cdots + 1) = 0$

$a = 1 \quad r = 1.003 \quad n = 360$

① correct GP = 0

$M = \dfrac{1500000 \times 1.003^{60}}{\frac{1.003^{360} - 1}{1.003 - 1}}$

$= \dfrac{1500000 \times 1.003^{60} \times 0.003}{1.003^{360} - 1}$

$= \$6819.68$ ①

Total repaid $= 360 \times 6819.68$
$= \$2455084.89$ ①

Interest $= 2455084.89 - 1500000$
$= \$955084.89$

5	Correct M/Interest paid
4	M = \$6819.68
3	$A_{360} = 0$
2	Develops A_2
1	correct values for n = 360, r = 0.3% p.m, P = 1 500 000

Question 27 (5 marks)

Use the following standard z-table for this question.

	First decimal place									
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

(a) What is $P(z \geq 1.8)$? **1**

$= 1 - P(z < 1.8)$ 1/r/w

$= 1 - 0.9641$ (From table)

$= 0.0359$

$\doteqdot 3.59\%$

(b) What is $P(-1.2 \leq z \leq 0.3)$? **2**

$= P(z \leq 0.3) - P(z \geq 1.2)$

$= 0.6179 - (1 - 0.8849)$

$= 0.5028$

$= 50.28\%$

2 | correct
1 | progress — correctly calculated $P(z \geq 1.2)$ / found a difference with correct $P(z<0.3)$ in front.

(c) The birth weight of babies is known to be normally distributed. According to data which covers Australian pregnancies between 1998 and 2007, the mean μ birthweight for boys was 3632 grams with a standard deviation σ of 430 grams. Using the table above, what is the approximate probability that a randomly selected newborn boy will weigh less than 3890 grams? **2**

$z_{score} = \frac{3890 - 3632}{430}$

$= 0.6$

$P(z < 0.6) = 0.7257$

$= 72.57\%$

2 | correct
1 | calculating z score

Question 28 (7 marks)

Consider the curve $y = 4x^2 - 2x^3$.

(a) Find the stationary points of the curve $y = 4x^2 - 2x^3$. Determine their nature. **3**

$\frac{dy}{dx} = 8x - 6x^2$

when $\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 8 - 12x$

$2x(4 - 3x) = 0$

$\therefore 2x = 0$ or $3x = 4$

$x = 0$ or $x = 4/3$

When $x = 0$, $y = 0$

$x = 4/3$, $y = 4 \times (4/3)^2 - 2(4/3)^3$

$= \frac{64}{27}$

$\therefore$ stationary points are $(0, 0)$ and $(4/3, \frac{64}{27})$

When $x = 0$ $\frac{d^2y}{dx^2} = 8 - 12 \times 0$

> 0 $\therefore (0, 0)$ is a minimum stationary point

When $x = 4/3$ $\frac{d^2y}{dx^2} = 8 - 12 \times \frac{4}{3}$

< 0

$\therefore (4/3, 64/27)$ is a maximum stationary point

3	Correct
2	Calculation of derivatives, correct x values and testing
1	Just derivatives and equating first derivative to 0

Question 28 continues on next page

Question 28 continued

(b) Show that there is an inflection point on the curve. 1

When $\frac{d^2y}{dx^2} = 0$ $8 - 12x = 0$

$x = 2/3$ 1 r/w

When $x = 2/3$ $y = 4 \times (2/3)^2 - 2(2/3)^3$

$= \frac{32}{27}$

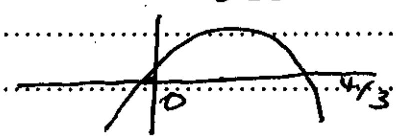
x	$1/3$	$2/3$	1
$\frac{d^2y}{dx^2}$	$8 - 12 \times 1/3$ > 0	0	$8 - 12$ < 0

Since the concavity changes either side $(2/3, \frac{32}{27})$ is a point of inflection

(c) For what interval is the curve $y = 4x^2 - 2x^3$ increasing? 1

When $8x - 6x^2 > 0$ i.e when $\frac{dy}{dx} > 0$

$2x(4 - 3x) > 0$ 1/r/w

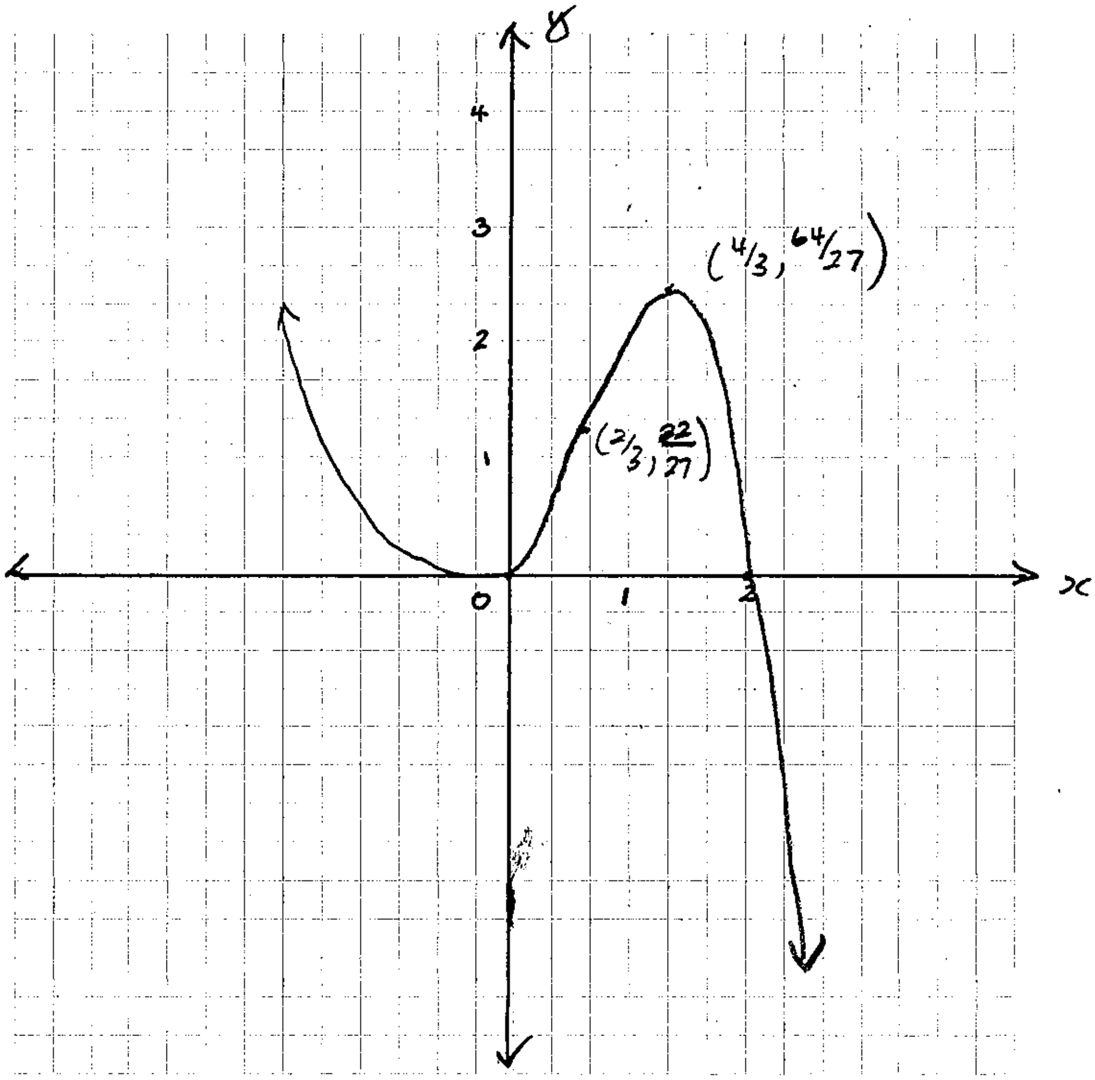


$0 < x < 4/3$

Question 28 continues on next page

Question 28 continued.

(d) Hence, sketch the graph of the curve $y = 4x^2 - 2x^3$. Clearly label the stationary points, the point of inflection and any intercepts with the axes. **2**



$y = 2x^2(2 - x)$

when $y = 0$ $x = 0$ or $x = 2$

2	correct
1	Mostly correct

Question 29 (3 marks)

(a) Differentiate $y = \dfrac{\ln x}{x}$. 1

$$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - \ln x}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

1 r/w

(b) Hence, evaluate $\displaystyle\int_e^{e^2} \frac{2 - \ln x^2}{x^2}\, dx$. 2

$$= \int_e^{e^2} \frac{2 - 2\ln x}{x^2}\, dx$$

$$= 2\int_e^{e^2} \frac{1 - \ln x}{x^2}\, dx$$

$$= 2\left[\frac{\ln x}{x}\right]_e^{e^2} \quad \text{from a)}$$

$$= 2\left\{\frac{\ln e^2}{e^2} - \frac{\ln e}{e}\right\}$$

$$= 2\left\{\frac{2}{e^2} - \frac{1}{e}\right\}$$

2	Correct
1	Correctly manipulating integral to obtain a)

Question 30 (4 marks)

Consider the probability density function f for a random variable X given by

$$f(x)=\begin{cases}\dfrac{a}{\sqrt{9+2x}} & 0\le x\le 8\\ 0 & \text{otherwise}\end{cases}$$

where a is a constant.

(a) Find the value of a. **2**

$a\int_0^8 (9+2x)^{-1/2}\,dx = 1$ 1

$a\left[\dfrac{(9+2x)^{1/2}}{\frac{1}{2}\times 2}\right]_0^8 = 1$

$25^{1/2} - 9^{1/2} = 1/a$

$2 = 1/a$

$a = 1/2$ 1

2	correct
1	correct first line.

(b) Find the cumulative distribution function, $F(x)$. **2**

$F(x) = \frac{1}{2}\int_0^x (9+2x)^{-1/2}\,dx \quad 0\le x\le 8$

$= \frac{1}{2}\left[(9+2x)^{1/2}\right]_0^x \quad 0\le x\le 8$

$= \frac{1}{2}\left\{(9+2x)^{1/2} - 9^{1/2}\right\}$

$F(x) = \frac{1}{2}\sqrt{9+2x} - 3/2$ 1

2	correct
1	correct integral and some progress

Question 31 (4 marks)

A particle moves along a straight line so that its displacement x metres to the right of a fixed point O is given by

$$x = 12\ln(t+2) - 2t + 5,$$

where the time t is measured in seconds.

(a) What is the initial position of the particle? Give your answer in exact form. **1**

When $t = 0$ $x = 12\ln 2 + 5$ 1 r/w

(or $\ln 2^{12} + 5$ or $\ln 4096 + 5$)

(b) Find the expression for the velocity of the particle at time t. **1**

$\dot{x} = 12 \times \dfrac{1}{t+2} - 2$

1 r/w

$\dot{x} = \dfrac{12}{t+2} - 2$

(c) Find the time when the particle is at rest. **1**

When $\dot{x} = 0$ $\dfrac{12}{t+2} - 2 = 0$

1 r/w

$\therefore 12 = 2(t+2)$

$t + 2 = 6$

$t = 4$ sec

(d) What happens to the acceleration eventually? **1**

$\ddot{x} = \dfrac{-12}{(t+2)^2}$

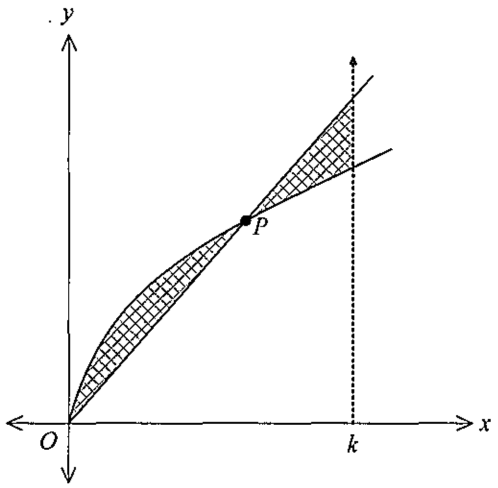
1/ww

As $t \to \infty$ $\ddot{x} \to 0$

approaches 0

Question 32 (4 marks)

The diagram below shows the area bounded between two curves, $y = \sqrt{x}$ and $y = x$.



(a) P is the point of intersection of the curves. Write down the coordinates of P. **1**

$\sqrt{x} = x$ when $x = 1$ $\therefore P(1, 1)$

1 ✓/w

Consider the region bounded by the curves, the line $x = 0$ and $x = k$, where $k > 1$.

(b) For what value of k will the shaded area to the left of point P be equal to the shaded area to the right of P? **3**

$\int_0^1 (\sqrt{x} - x)\, dx$

$= \int_0^1 (x^{1/2} - x)\, dx$

$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1$

$= \frac{2}{3} - \frac{1}{2}$

$= 1/6$ 1

$\int_1^k (x - x^{1/2})\, dx = 1/6$

$\left[\frac{x^2}{2} - \frac{2x^{3/2}}{3}\right]_1^k = 1/6$

$\frac{k^2}{2} - \frac{2k^{3/2}}{3} - \left(\frac{1}{2} - \frac{2}{3}\right) = 1/6$ 1

$\frac{k^2}{2} - \frac{2k^{3/2}}{3} = 0$

$3k^2 - 4k^{3/2} = 0$

$k^{3/2}(3k^{1/2} - 4) = 0$

$\therefore k^{3/2} = 0$ or $k^{1/2} = 4/3$

$\therefore k = 0$ or $k = 16/9$ 1

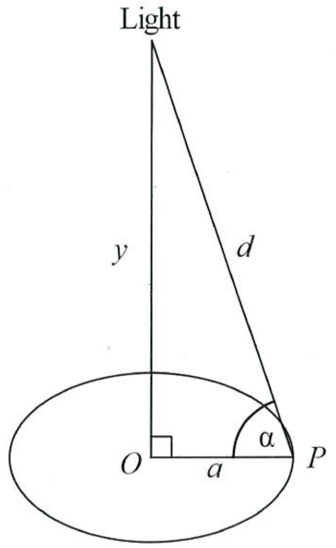
Not a solution since $k > 1$

3	Correct
2	Calculation of area between 0 and 1 and progress towards other part
1	Area between 0 and 1

Question 33 (5 marks)

A light is to be placed over the centre of a circle, radius a units. The intensity I of the light is proportional to the sine of the angle α at which the rays strike the circumference of the circle, divided by the square of the distance d from the light to the circumference of the circle,

i.e. $I = \dfrac{k \sin \alpha}{d^2}$, where k is a positive constant.



NOT TO SCALE

(a) Show that $I = \dfrac{ky}{\left(y^2 + a^2\right)^{\frac{3}{2}}}$. **2**

In $\triangle LOP$ $\sin\alpha = y/d$ $d^2 = a^2 + y^2$ by Pythagoras' Theorem

$\therefore I = \dfrac{k \times y/d}{a^2 + y^2}$

$= \dfrac{ky}{d(a^2 + y^2)}$

$= \dfrac{ky}{\sqrt{a^2+y^2}\,(a^2 + y^2)}$

$I = \dfrac{ky}{(a^2 + y^2)^{3/2}}$

2	correct
1	Correct calculation of $\sin\alpha$, d^2 and substituting into given formula

↑ Need to show progress toward answer

Question 33 continues on next page

Question 33 continued

(b) Find the best height for the light to be placed over the centre of the circle in order to provide maximum illumination to the circumference. **3**

$$\frac{dI}{dy} = \frac{(y^2+a^2)^{3/2} \times k - ky \times \frac{3}{2}(y^2+a^2)^{1/2} \times 2y}{(y^2+a^2)^3}$$

$$= \frac{k(y^2+a^2)^{3/2} - 3ky^2(y^2+a^2)^{1/2}}{(y^2+a^2)^3}$$

$$= \frac{k(y^2+a^2)^{1/2}\left((y^2+a^2) - 3y^2\right)}{(y^2+a^2)^3}$$

$$= \frac{k(a^2-2y^2)}{(y^2+a^2)^{5/2}}$$

1

When $\frac{dI}{dy} = 0$ $\quad k(a^2 - 2y^2) = 0$

$$2y^2 = a^2$$

$$y^2 = a^2/2$$

$$y = \frac{a}{\sqrt{2}}\ ;\ y > 0$$ 1

Since $a > 0$

or test with 2nd derivative

y	$\frac{a}{2\sqrt{2}}$	$a/\sqrt{2}$	$\frac{2a}{\sqrt{2}}$
$\frac{dI}{dy}$	$k(a^2 - \frac{2a^2}{8})$ > 0	0	$k(a^2 - \frac{8a^2}{2})$ < 0

1

$\therefore y = a/\sqrt{2}$ is the greatest height

3	correct by showing answer is a maximum.
2	correct derivative and determining a value for y. 1 error in process
1	Attempted derivative.

End of Examination